Multi-Robot Ergodic Trajectory Optimization with Relaxed Periodic Connectivity

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***Abstract*— This paper considers a multi-robot trajectory planning problem with inter-robot connectivity maintenance for information gathering. Given an information map in the form of a distribution over the workspace, ergodic search plans trajectories, along which, the time spent in any region is proportional to the amount of information in that region, and can balance between exploration and exploitation. Existing ergodic search rarely considers the limited communication range among robots or connectivity maintenance, and this paper takes a step to fill this gap. Besides, multi-robot connectivity maintenance was studied a lot, including continual, periodic, intermittent connectivity, etc. Naively combining these methods with ergodic search may prevent the planner from finding high-quality ergodic trajectories or lead to poor connectivity among the robots. To handle the challenge, this paper adapts an intermittent connectivity maintenance strategy to the ergodic search framework, and develops a two-phase trajectory planning approach utilizing the augmented Lagrangian method. Our simulation and real drone experiments show that under the same connectivity maintenance requirement, our approach plans trajectories that are about 10 times better than the baselines in terms of the ergodic metric.**

I. INTRODUCTION

This paper studies a multi-robot trajectory planning problem for information gathering subject to connectivity maintenance constraints, which arise in applications such as exploration [1], and search and rescue [2]. Using an information map to describe the prior knowledge in the form of a distribution over the area to be searched, this paper aims to plan trajectories for the robots to collect information within this map and establish the robot-robot connection when needed to exchange information. Existing approaches for information-gathering range from spatial decomposition [3], [4] , which uniformly covers the area, to information-theoretic approaches [5], [6], which greedily direct the robot to the next location with the highest information gain. Different from these methods, ergodic search [7], [8] provides an approach that can inherently balance between exploration (visiting all possible locations for new information) and exploitation (greedily searching high-information areas), which plans trajectories by optimizing an ergodic metric so that the time spent in any region is proportional to the amount of information in that region.

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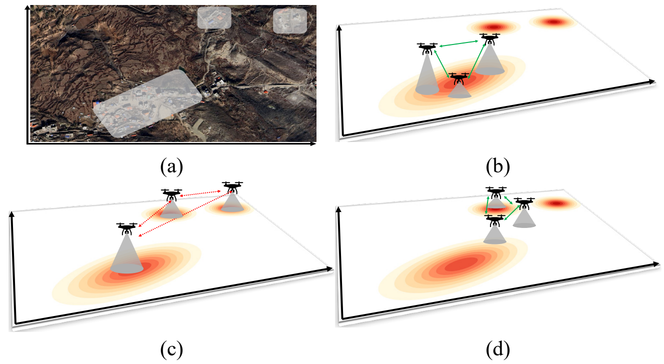


Fig. 1. A motivating example. (a): Mountainous area with three regions to be searched. (b): Drones are connected while gathering information. (c): Drones disconnect to gather information in different regions. (d): Drones reconnect to share the collected information. The planner needs to handle both connectivity maintenance and information gathering.

Although ergodic search has been investigated from various perspectives [8]–[15], most of them either ignore the limited connectivity among robots [12], [13], assuming all robots are connected with a central communication hub at all times [10], [11], or require all robots to stay connected with pre-determined and fixed topology [14]. [15] implicitly guarantees the connectivity asymptotically through the decentralized ergodic control algorithm. This paper aims to let the robots determine flexibly when and where to establish connections during the ergodic trajectory optimization.

To achieve this goal, we investigate how to adapt the existing intermittent connectivity maintenance approach within the ergodic search framework, and propose a method named **E**rgodic **S**earch with Relaxed **P**eriodic **C**onnectivity (ESPC), which determines the location and time for inter-robot connection during the ergodic trajectories optimization. Fig. 1 shows the motivation example. First, ESPC introduces the notion of relaxed periodic connectivity, an adapted version of intermittent connectivity for continuous trajectory optimization, where any pair of robots should have their distance smaller than the communication range at least once within each relaxed period, which constitutes the optimization objective, i.e., connectivity cost. We then approximate this binary connectivity cost using the Sigmoid function so that the gradient can be obtained for trajectory optimization. Second, ESPC uses a two-phase solving approach based on the augmented Lagrangian method so that the resulting trajectories can converge to a local minimum with a low ergodic metric while maintaining connectivity.

To verify our approach, we combine ergodic search with two existing connectivity maintenance strategies in the literature [16], [17] as the baselines, which let all robots meet at certain locations periodically, i.e., at fixed time intervals. We then compare them in both simulations and drone experiments under various settings. Results show that, under the same connectivity maintenance requirement, our approach plans better trajectories whose ergodic metrics are about 10 times smaller than the baselines. Both Gazebo simulation and drone experiments in a lab setting verify that the planned ergodic trajectories of the robots are executable while intermittent connectivity is maintained.

A. Related Work

Connectivity maintenance has been studied a lot [18]–[21] and combined with information gathering [16], [22], [23], task planning [17], or environment exploration [24]–[26]. Some of them require all robots to stay connected at all times (i.e., continual connectivity) [19], [20] , which is suitable for communication critical missions, but usually hinders robots from dispersing to gather information quickly, especially in a large workspace with distributed information. Additionally, the informative path planning method with periodic connectivity in [23] requires all robots to regain connectivity at a pre-defined fixed interval. However, the connectivity-constrained feasible paths are planned through sampling or enumerating rather than optimizing with nonlinear robot dynamics. Besides, intermittent connectivity [21] is widely studied, which enables robots to meet at some locations over time, infinitely often. The communication schedules with the sampling-based path planning method [22] or predefined communication points set [17] were also investigated. Other studies on connectivity maintenance include choosing rendezvous locations [26], finding a ground robot [25], and planning reconnection paths [16].

Despite extensive research on intermittent connectivity maintenance, most existing methods [17], [22], [23] rely on sampling-based approaches for connection establishment, making them unsuitable for ergodic trajectory optimization problems typically solved through iterative gradient descent. Furthermore, some optimization-based studies focus on maximizing the Fiedler value of the connectivity graph, primarily designed for continual connectivity scenarios [20].

II. Preliminary

A. Workspace and Robot Dynamics

Let the index set $I_N = \{1, 2, \cdots, N\}$ denote a set of N robots. Let $\mathcal{W} = [-L'_1, L''_1] \times \cdots \times [-L'_\nu, L''_\nu] \subset \mathbb{R}^\nu, L'_\nu, L''_\nu \in \mathbb{R}^+, \nu \in \{2, 3\}$ denote a ν-dimensional workspace that is to be explored by the robots, where $L_\nu = ||L''_\nu + L'_\nu||$ is the length of the workspace along the ν-th dimension. This paper uses a superscript i over a variable to indicate the robot associated with the variable. For robot $i \in I_N$, let $x^i : [0, T] \to \mathcal{X} \subseteq \mathbb{R}^n, n \in \mathbb{Z}^+, n \geq \nu$ denote the state trajectory within the time horizon $T \in \mathbb{R}^+$, and let $u^i : [0, T] \to \mathcal{U} \subseteq \mathbb{R}^m, m \in \mathbb{Z}^+$ represent the control trajectory. All robots are homogeneous in the sense that each robot $i \in I_N$ has the same deterministic dynamics $\dot{x}^i(t) = f(x^i(t), u^i(t))$. Additionally, let $q^i : [0, T] \to \mathcal{W}$ denote the corresponding trajectory of robot i in the workspace, and let $g : \mathcal{X} \to \mathcal{W}$ denote a map from a state to the corresponding location in the workspace, i.e., $q^i(t) = g(x^i(t))$.

Let $x = (x^1, \cdots, x^N)$ denote the joint state trajectory of all robots. Similarly, let $u = (u^1, \cdots, u^N)$ denote the joint control trajectory. Let $q = g(x) = (g(x^1), \cdots, g(x^N)) = (q^1, \ldots, q^N)$ denote the joint trajectory in the workspace. Finally, let $\dot{x}(t) = F(x(t), u(t))$ denote all robots' dynamics.

B. Ergodic Search

Let $\phi^i(w) : \mathcal{W} \to \mathbb{R}_0^+$ denote an information map to be explored by robot $i \in I_N$, which is a known time-invariant probability distribution over the workspace with $\int_\mathcal{W} \phi^i(w)dw = 1$ and $\phi^i(w) \geq 0, \forall w \in \mathcal{W}$. Intuitively, $\phi^i(w)$ provides the information density at each location in the workspace. Let $\delta(w)$ denote the Dirac delta function, which satisfies $\int_{-\infty}^{+\infty} \delta(w)dw = 1$, $\delta(0) = +\infty$ and $\delta(w) = 0$ when $w \neq 0$. Let $c(w, q^i)$ denote the time-averaged statistics of the trajectory q^i, which is defined as follows.

$$c(w, q^i) = \frac{1}{T} \int_0^T \delta(w - q^i(t))dt \tag{1}$$

Then, the ergodic metric [7] can measure the coverage of the information map ϕ^i by the trajectory q^i:

$$\mathcal{E}^i(\phi^i, q^i) = \sum_{\mathbf{k} \in \mathcal{K}} \Lambda_\mathbf{k} (c^i_\mathbf{k} - \phi^i_\mathbf{k})^2 \tag{2}$$

$$= \sum_{\mathbf{k} \in \mathcal{K}} \Lambda_\mathbf{k} \left(\frac{1}{T} \int_0^T F_\mathbf{k}(q^i(t))dt - \int_\mathcal{W} \phi^i(w) F_\mathbf{k}(w)dw \right)^2$$

where $c^i_\mathbf{k}$ and $\phi^i_\mathbf{k}$ are the Fourier coefficients of $c(w, q^i)$ and $\phi^i(w)$, respectively. $\mathbf{k} = [k_1, \cdots, k_\nu] \in \mathcal{K}$ is the frequency vector, and $\mathcal{K} \subset \mathbb{N}^\nu$ represents the set of considered frequencies. $F_\mathbf{k}(w) = \frac{1}{h_\mathbf{k}} \prod_{o=1}^{\nu} \cos \frac{\pi k_o w_o}{L_o}$ is the cosine basis function with the normalization term $h_\mathbf{k}$ [7]. $\Lambda_\mathbf{k} = (1 + \| \mathbf{k} \|_2^2)^{-(\nu+1)/2}$ is the weight of each Fourier coefficient. The multi-robot ergodic search problem is formulated as:

Problem 1 (Multi-Robot Ergodic Search)**.**

$$\min_{x,u} \quad \sum_{i \in I_N} \mathcal{E}^i(\phi^i, q^i) \tag{3a}$$

$$\text{s.t.} \quad x(0) = x_0 \tag{3b}$$

$$\dot{x} = F(x, u) \tag{3c}$$

$$||q^i(t) - q^j(t)|| \geq \varepsilon_d, i \neq j \in I_N, t \in [0, T] \tag{3d}$$

$$x^i \in \mathcal{X}, u^i \in \mathcal{U}, \forall i \in I_N \tag{3e}$$

where the objective (3a) is the sum of all robots' ergodic metrics, (3b) specifies the initial states of all robots, (3c) represents the robots' dynamics constraints, (3d) ensures robot-robot collision avoidance with a tolerance of $\varepsilon_d \in \mathbb{R}^+$, and (3e) bounds the state and control.

Besides, when the information maps of all the robots are the same, i.e., $\phi^1 = \cdots = \phi^N$, we use $\bar{\phi}$ to denote this common information map. The objective for Problem 1 in this case, can be written as follows.

$$\mathcal{E}(\bar{\phi}, q) = \sum_{\mathbf{k} \in \mathcal{K}} \Lambda_\mathbf{k} (\bar{c}_\mathbf{k} - \bar{\phi}_\mathbf{k})^2 \tag{4}$$

where $\bar{c}_{\mathbf{k}} = \frac{1}{N}\sum_{i \in I_N} c^i_{\mathbf{k}}$ is the average of all robots' $c^i_{\mathbf{k}}$ and $\bar{\phi}_{\mathbf{k}}$ is the Fourier decomposition of $\bar{\phi}(w)$.

III. Relaxed Periodic Connectivity

A. Relaxed Connection Period

Let $d_{ij}(t) = ||q_i(t) - q_j(t)||_2$ denote the Euclidean distance between any two robots $i, j \in I_N, i \neq j$ at time t. Let $R_c \in \mathbb{R}^+$, a known constant number, denote the connection range. When any two robots are within the range (i.e., $d_{ij}(t) \leq R_c$), these two robots are said to be *connected* and can communicate with each other. In addition, let the binary variable $s(i,j,t) \in \{0,1\}$ denote whether the robots i and j are connected at time t ($s(i,j,t) = 1$) or not ($s(i,j,t) = 0$).

Let the time horizon T be evenly divided into $N_p \in \mathbb{Z}^+$ periods, where each period is of length $T_p = T/N_p$. A time point t belongs to the p-th period if $(p-1)T_p < t \leq pT_p$, $p \in I_p = \{1, 2, \cdots, N_p\}$. Simply requiring that every pair of robots connect at any time during each period may result in a situation where two robots connect near the end of the p-th period and then again at the beginning of the $(p+1)$-th period. The interval between these two subsequent connections can be small, which is undesired. Therefore, we introduce a user-defined parameter $\tau \in [0,1]$ and require that any pair of robots connect within the time range $T_{p,\tau} := [(p-\tau)T_p, pT_p]$ for the p-th period. We call the range $T_{p,\tau}$ a *relaxed connection period*. When $\tau = 0$, $T_{p,\tau}$ becomes a singleton time point and leads to the notion of periodic connectivity in the literature [23], which requires every pair of robots to connect at pre-determined intervals. Finally, note that the relaxed periodic connectivity does not require all robots to be connected at the same time point. The motivation behind relaxed connection periods is to let the robots connect intermittently with certain intervals for information sharing while avoiding imposing hard constraints at fixed connection time points, which allows the robots to determine when to connect flexibly.

B. Connectivity Cost

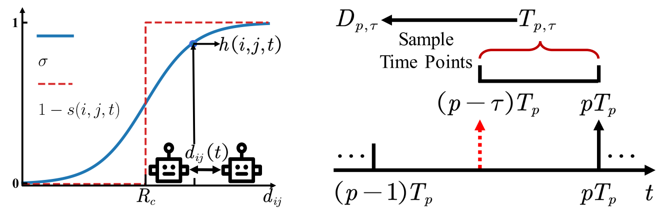
Robots $i, j \in I_N, i \neq j$ are connected in the p-th period if there exists a time point $t \in T_{p,\tau}$ such that $s(i,j,t) = 1$. The connectivity cost $c_{con}(i,j,p)$ between robot i and j of the p-th relaxed connection period is defined as:

$$c_{con}(i,j,p) = \begin{cases} 0, & \text{if } s(i,j,t) = 1, \exists t \in T_{p,\tau} \\ 1, & \text{other} \end{cases} \tag{5}$$

Any pair of robots are connected in the p-th period if there exists a set of time points $t_{i,j} \in T_{p,\tau}, i, j \in I_N, i \neq j$ in the p-th period such that robots i, j are connected at time $t_{i,j}$. For all robots and all periods, the total connectivity cost C_{con} is defined as:

$$C_{con} = \sum_{i \in I_N} \sum_{j \in I_N, j \neq i} \sum_{p \in I_p} c_{con}(i,j,p) \tag{6}$$

The ergodic search problem with relaxed periodic connectivity is formulated as follows.



(a) (b)

Fig. 2. An illustration of the connectivity cost for the p-th period. (a): The Sigmoid function σ converts the discrete value $1 - s(i,j,t)$ to a continuous value between 0 and 1 based on the distance d_{ij}. (b): $D_{p,\tau}$ in Eq. (9) stores the sampled time points from $T_{p,\tau}$, where τ constrains the length of $T_{p,\tau}$.

Problem 2 (Multi-Robot ESPC Problem)**.**

$$\min_{x,u} \quad C_{con} \tag{7a}$$
$$\text{s.t.} \quad \text{(3b), (3c), (3d), (3e)} \tag{7b}$$
$$\mathcal{E}^i(\phi^i, q^i) \leq \mathcal{E}^i_m, \forall i \in I_N \tag{7c}$$

where (7b) inherits from Problem 1, and (7c) constrains the ergodic metric within tolerable limits $\mathcal{E}^i_m$. Note that when the information map is $\bar{\phi}$, (7c) can be written as $\mathcal{E}(\bar{\phi}, q) \leq \mathcal{E}_m$.

Remark 1. *Combining ergodic search with connectivity maintenance involves optimizing two objectives (often competitive due to dispersed information in the workspace): minimizing the ergodicity and the total connectivity cost. Instead of combining them into a single or bi-objective formulation, we approach it as a constrained optimization problem, treating C_{con} as the objective (soft constraint) and ergodicity as the constraint. The intuition behind this choice is as follows. Ergodic trajectory optimization often yields multiple local minima with similar ergodicity. The constrained optimization formulation allows our planner to explore different local minima so that the connections among the robots are established while the ergodicity is maintained at a low value. In other words, the goal is to find an alternative solution under the constraint (7c) with limits $\mathcal{E}^i_m$ (discussed in Sec. V-A.4).*

IV. Optimization Algorithm

The binary variables $c_{con}(i,j,p)$ in the objective of Problem 2 are non-differentiable and prevent the use of trajectory optimization techniques. We first use a continuous function to smooth $s(i,j,t)$, then describe the trajectory optimization.

A. Continuous Approximation of Connectivity Cost

As shown in Fig. 2a, this paper uses the Sigmoid function $\sigma : \mathbb{R} \rightarrow \mathbb{R}$ to approximate $1 - s(i,j,t)$ based on the distance $d_{ij}(t)$ between robots i, j. Specifically, let

$$h(i,j,t) = \sigma(d_{ij}(t)) = \frac{1}{1 + e^{-\rho(d_{ij}(t) - R_c)}} \tag{8}$$

where $\rho \in \mathbb{R}^+$ is a constant coefficient affecting the shape of the Sigmoid function, indicating the sensitivity to $d_{ij}(t)$ around R_c. The continuous range of $h(i,j,t)$ is $[0,1]$, taking value 0 when $d_{ij}(t) \ll R_c$ and 1 when $d_{ij}(t) \gg R_c$.

Within the relaxed period $T_{p,\tau}$, a pair of robots is connected if there exists at least a time point $t \in T_{p,\tau}$ such that

Algorithm 1 Two-Phase Approach

1: $I_N, I_p, \tau \leftarrow$ Initial parameters
2: $x = x_{init}, u = \mathbf{0} \leftarrow$ Initial state and control ▷ **Phase 1**
3: Solve **Problem 1** with maximum iteration K using Alg. 2
4: $x_\star, u_\star, \mathcal{E}_\star^i, i \in I_N \leftarrow$ Get solution and metric
5: $\mathcal{E}_m^i, i \in I_N \leftarrow$ Initial ergodic metric limits ▷ **Phase 2**
6: Solve **Problem 2** with maximum iteration K using Alg. 2 with initial guess $x_\star, u_\star$, and limits $\mathcal{E}_m^i, i \in I_N$
7: $x_c, u_c, \mathcal{E}_c^i, i \in I_N \leftarrow$ Get solution and metric

Algorithm 2 Iterative Optimization

1: $\mathbf{x}, \mathbf{u}, \mathcal{O}, \lambda, \mu, \hat{\alpha}, r, \varepsilon_C, \hbar, \leftarrow$ Initial parameters
2: $C(\mathbf{x}, \mathbf{u}) = \sum_{k \in I_{N_T}} ||h_k||_2 + \sum_{i \in I_N} ||c_i||_2$
3: **while** $C(\mathbf{x}, \mathbf{u}) \geq \varepsilon_C$ **do** ▷ With maximum iteration K
4: $\quad \mathbf{x}', \mathbf{u}' \leftarrow \arg\min_{\mathbf{x},\mathbf{u}} L_r(\mathbf{x}, \mathbf{u}, \lambda, \mu)$ by $\mathcal{O}$, see (10)
5: $\quad$ **if** $C(\mathbf{x}, \mathbf{u}) - C(\mathbf{x}', \mathbf{u}') \geq \hbar$ **then**
6: $\quad\quad$ Update $\lambda_k, k \in I_{N_T}$ and $\mu_i, i \in I_N$ through (11)
7: $\quad$ **else**
8: $\quad\quad$ Update penalty $r \leftarrow \hat{\alpha} \cdot r$
9: $\quad \mathbf{x} \leftarrow \mathbf{x}', \mathbf{u} \leftarrow \mathbf{u}'$

$d_{ij}(t) \leq R_c$. We formulate this condition by first uniformly sampling a set of time points $D_{p,\tau}$ from $T_{p,\tau}$ (refer to Fig. 2b), and then taking the product of the $h(i,j,t)$ value at those sampled time points. Then, the connectivity cost (5) can be approximated by a continuous function $c_{con}^h(i,j,p)$:

$$c_{con}^h(i,j,p) = \prod_{t \in D_{p,\tau}} h(i,j,t) \tag{9}$$

If there is at least a time point $t \in T_{p,\tau}$ such that $d_{ij}(t) \leq R_c$, the corresponding $h(i,j,t)$ takes a near-zero value and can lower the value of $c_{con}^h(i,j,p)$, resulting in a low connectivity cost. Finally, similar to (6), let $C_{con}^h = \sum c_{con}^h(i,j,p)$ denote the approximated total cost.

B. Two-Phase Approach

Directly solving Problem 2 with a random initial guess may lead to dynamically infeasible trajectories with high connectivity cost, or make the computation time-consuming (Refer to Sec. V-C.5). We thus develop a two-phase approach, where the first phase computes a dynamically feasible ergodic trajectory $x_\star$ by solving Problem 1 that ignores the connectivity maintenance requirement. Then the second phase uses $x_\star$ as the initial guess to solve Problem 2 to reduce the connectivity cost while ensuring that the trajectories are still dynamically feasible and the ergodic metric stays within tolerance $\mathcal{E}_m^i$.

Specifically, the first phase (Line 2-4 in Alg. 1) solves Problem 1 to obtain the joint control trajectory $u_\star = (u_\star^1, \cdots, u_\star^N)$, joint state trajectory $x_\star = (x_\star^1, \cdots, x_\star^N)$, and the corresponding ergodic metric $\mathcal{E}_\star^i, i \in I_N$, where the initial guess is set with $x(t) = x_0, t \in [0,T]$ and a zero control input $u = \mathbf{0}$ in this paper. Note that other initial guesses can also be used for phase 1, e.g., random controls. Phase 2 (Line 5-7 in Alg. 1) sets $x_\star, u_\star$ as the initial guess to solve Problem 2. In this phase, the augmented Lagrangian method (ALM) incorporates the ergodic constraint (7c) into the Augmented Lagrangian Function.

C. Optimization Method

On Lines 3 and 6 of Alg. 1, any nonlinear optimizer can be used. The optimization method used in this paper converts the constrained optimization problem into an unconstrained one based on ALM. To simplify the presentation, we omit the constraints (3b,3d,3e) of Problem 2 when presenting ALM, and explain the ideas via the dynamics constraints (3c) and the ergodic metric constraints (7c). Constraints (3b,3d,3e) are handled similarly in our implementation.

1) Numerical Integration of the Dynamics: We numerically integrate the dynamics to convert the trajectory optimization to a nonlinear optimization problem over a finite number of trajectory points. Let $\delta_t = T/N_T$ denote the time step size, where N_T is the number of time steps. Let $\mathbf{x} = (x(0); \cdots; x(N_T \cdot \delta_t)), \mathbf{u} = (u(0); \cdots; u((N_T - 1) \cdot \delta_t))$ denote a matrix that stacks all discrete states and controls, and $\mathbf{x}_k, \mathbf{u}_k$ denote the k-th vector of $\mathbf{x}, \mathbf{u}$. This paper uses the Euler method for integration, i.e., $\mathbf{x}_{k+1} = \mathbf{x}_k + \delta_t \cdot F(\mathbf{x}_k, \mathbf{u}_k), \forall k \in I_{N_T} = \{0, 1, \cdots, N_T - 1\}$, and the control input $\mathbf{u}_k$ is a constant between two subsequent time steps (often referred to as the zero-order hold).

2) Augmented Lagrangian Function: Let $h_k(\mathbf{x}, \mathbf{u}) = \mathbf{x}_{k+1} - (\mathbf{x}_k + \delta_t \cdot F(\mathbf{x}_k, \mathbf{u}_k)), k \in I_{N_T}$ denote the k-th dynamic (equality) constraint. Let $c_i(\mathbf{x}, \mathbf{u}) = \mathcal{E}^i(\phi^i, q^i) - \mathcal{E}_m^i, i \in I_N$ denote the i-th ergodic metric (inequality) constraint. Based on the formulation of Problem 2, the augmented Lagrangian function is formulated as follows [27].

$$\begin{aligned} L_r(\mathbf{x}, \mathbf{u}, \lambda, \mu) = & C_{con}^h + \sum_{k \in I_{N_T}} \{\lambda_k h_k + \frac{r}{2} h_k^2\} \\ & + \sum_{i \in I_N} \{\mu_i (c_i + s_i) + \frac{r}{2}(c_i + s_i)^2\} \end{aligned} \tag{10}$$

where λ_k and $\mu_i \geq 0$ are the Lagrange multipliers related to the k-th equality constraint h_k, and the i-th inequality constraint c_i, and $r \geq 0$ is a penalty factor that penalizes the violation of the constraints with second-order terms. Each λ_k is a vector corresponding to the state $\mathbf{x}_k$. Here, $s_i = \max\{-\mu_i/r - c_i, 0\}$ is a slack variable [27], which is zero when the constraint is active ($c_i > 0$) and has no effect on the optimum of the objective when the constraint is inactive. During the optimization, the penalty terms in (10) are increased for the violation of the constraints through $r \leftarrow \hat{\alpha} \cdot r$, where $\hat{\alpha} \in [1, +\infty)$.

3) Iterative Optimization: On Line 4 of Alg. 2, the objective (10) is minimized using optimizer $\mathcal{O}$. If $\mathbf{x}, \mathbf{u}$ are updated only once after the gradient of the objective (10) with respect to $\mathbf{x}, \mathbf{u}$ is computed, it may lead to slow convergence or even non-convergence. Besides, computing the gradient frequently and then updating $\mathbf{x}, \mathbf{u}$ with a fixed step size may trap the optimization into a local optimum. To remedy these difficulties, we use the Limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS) algorithm [28] with line search as the optimizer $\mathcal{O}$ [29]. By approximating the Hessian of the objective (10), L-BFGS

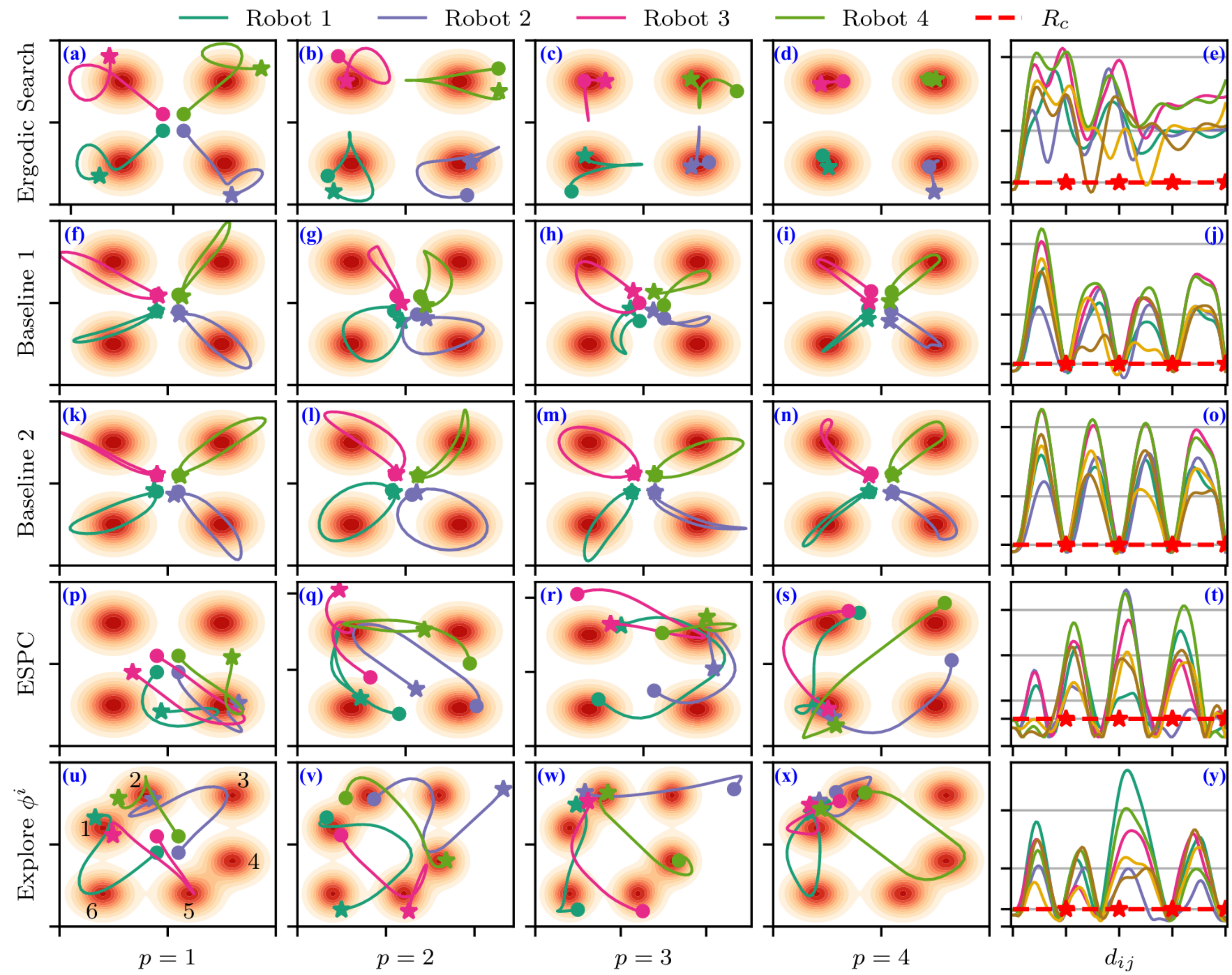


Fig. 3. Robots explore the common information map $\bar{\phi}$ using different methods (Row 1-4: Ergodic Search, Baseline 1: robots connect within the same predefined area periodically, Baseline 2: robots connect periodically considering minimal reconnection paths, ESPC: robots connect intermittently within different areas) and the different maps ϕ^i with our ESPC (Row 5). In subfigure (u), as indicated by the numbered labels, each robot has its information map, which is a mixture of Gaussian: $\phi^1 : 1, 6$; $\phi^2 : 2, 3$; $\phi^3 : 1, 5$; $\phi^4 : 2, 4$. The first four columns represent the trajectories of four connection periods, where the dot and asterisk represent the start and end of the trajectory during the p-th period, respectively. The last column shows the distance between all pairs of robots during all periods, i.e., $d_{ij}(t), t \in [0, T]$. The asterisk and red dash line represent the end of each period and the connection range, respectively. $d_{ij}(t)$ is used to determine connectivity; for example, $d_{ij}(t) \leq R_c$ indicates that robots i and j are connected at time t.

provides an accurate update direction and is able to expedite the convergence.

Here, $C(\mathbf{x}, \mathbf{u})$ on Line 2 in Alg. 2 is to compute the sum of equality and inequality constraints error for the optimization problems. The multipliers λ_k, μ_i, and the penalty factor r are updated based on the decreasing value of $C(\mathbf{x}, \mathbf{u})$, when it's greater than a user-specified tolerance $\hbar \in \mathbb{R}^+$ as [27]:

$$\lambda_k \leftarrow \lambda_k + r \cdot h_k, \quad \mu_i \leftarrow \max\{0, \mu_i + r \cdot c_i\} \tag{11}$$

The initial values of λ_k, μ_i, r are $\mathbf{0}, 1, 1$ in this paper, respectively. Although the objective C^h_{con} and constraints of Problem 2 are optimized simultaneously through the function (10), the constraints must be strictly satisfied for trajectory optimization problems. The optimization terminates when $C(\mathbf{x}, \mathbf{u}) < \varepsilon_C$ in this paper, i.e., the constraints error is within a hyperparameter $\varepsilon_C \in \mathbb{R}^+$.

V. Experimental Result

A. Experiment Settings

1) Robot Dynamics: Our tests consider quadrotors. The state of robot i is $x^i = (q^i, \overline{\mathbf{Rot}}^i, v^i, \omega^i)$, where q^i, v^i, ω^i denote the position, velocity, and angular velocity. $\overline{\mathbf{Rot}}^i$ represents the flattened vector of the rotation matrix $\mathbf{Rot}^i \in SO(3)$. Additionally, the control is defined as the forces provided by the four motors, i.e., $u^i = (u^i_1, u^i_2, u^i_3, u^i_4)$, which are used to generate the total thrust and moments. The dynamic equations are the same as those defined in [30].

2) Experiment Parameters: We consider a workspace of size $\mathcal{W} : [-5, 5] \times [-5, 5] \times [0, 3]$ m^3. The physical parameters of the quadrotor, such as mass, inertia, and others, match those defined in Crazyswarm2 [31]. Besides, each robot i has state and control limits $u^i(t) \in [\mathbf{0}, \mathbf{0.1}]$ N, velocity $v^i(t) \in [\mathbf{0}, \mathbf{0.8}]$ m/s, and angular velocity $\omega^i(t) \in$

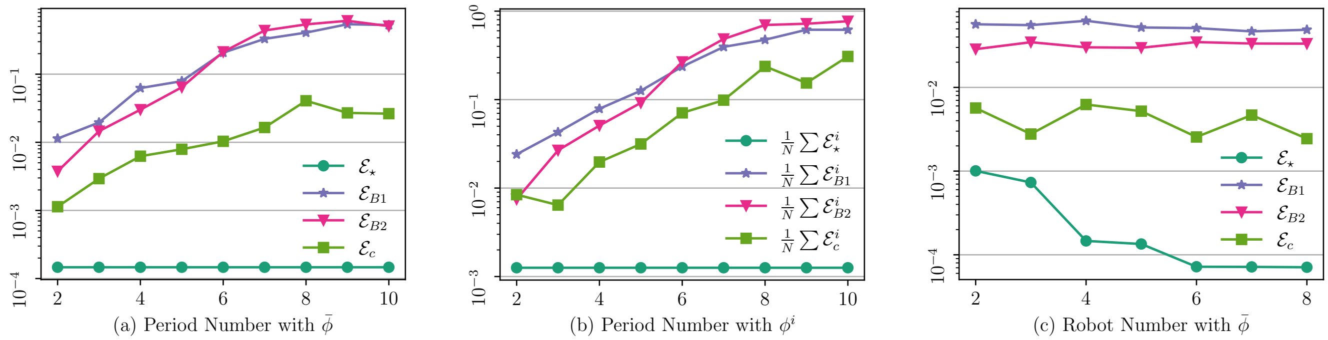


Fig. 4. The ergodic metrics $\mathcal{E}_\star$ (Ergodic search without connectivity maintenance), $\mathcal{E}_{B1}$ (Baseline 1), $\mathcal{E}_{B2}$ (Baseline 2), and $\mathcal{E}_c$ (ESPC) with varying period number N_p and varying number of robots N. (a): All robots explore the same information map $\bar{\phi}$ as in Row 1-4 (a-t) of Fig. 3. (b): Each robot explores its map $\phi^i, i \in I_N$ as in Row 5 (u-y) of Fig. 3. (c): With $N_p = 4$, the ergodic metric as the number of robots varies.

$[\mathbf{0}, (0.2, 0.2, 0.4)]$ rad/s, where bold letters indicate vectors. The initial state is defined as $x^i_0 = [q^i_0, \overline{\mathbf{E}}_3, \mathbf{0}, \mathbf{0}]$, where q^i_0 is the initial location and $\mathbf{E}_3$ is the unit matrix of order 3. Other parameters are set as follows: frequency set: $\mathcal{K} = [0, \cdots, 8] \times [0, \cdots, 8] \times \mathbf{0}$, connection range $R_c = 1.5$ m, collision avoidance tolerance ε_d : 0.5 m, constant in (8): ρ : 2.0, time related constants τ : 0.5 (see Fig. 2b), T : 60 s, δ_t : 0.5 s, factor in Alg. 1 and 2 $K : 50, \hat{\alpha} : 1.5, \varepsilon_C = 0.01T/\delta_t \cdot N, \hbar = \varepsilon_C$.

3) Additional Height Constraint: The workspace dimension is $\nu = 3$ in this paper, for the height (i.e., z dimension), we add a constraint $(q_z - H)^2 \leq \varepsilon_H$, where q_z, $H = 1.0$ m, and $\varepsilon_H = 10^{-3}$ m denote the trajectory height, desired height, and height deviation, respectively. This constraint ensures q_z (the height of the robots) is around a constant H. Specifically, it constrains the robots to fly within $H \pm \sqrt{\varepsilon_H}$ in both simulation and hardware experiments.

4) Selection of $\mathcal{E}_m$: The parameter $\mathcal{E}^i_m, i \in I_N$ can be set in different ways. One possibility is to inflate the ergodic metric $\mathcal{E}^i_\star$ obtained from the first phase by a factor $(1 + k_m), k_m \geq 0$, i.e., $\mathcal{E}^i_m = (1 + k_m)\mathcal{E}^i_\star$. In our experiments, our goal is to compare different methods, and we set $\mathcal{E}^i_m$ as follows. Let $\mathcal{E}^i_B$ denote the ergodic metric obtained by a baseline method discussed in Sec. V-B, $\mathcal{E}^i_m$ for robot i is then set as $\mathcal{E}^i_m = \text{Clip}(\gamma \cdot \mathcal{E}^i_B, \mathcal{E}^i_\star, \mathcal{E}^i_B)$, where $\gamma \in [0, 1]$, and Clip restricts the value of the variable $\gamma \cdot \mathcal{E}^i_B$ to the interval $[\mathcal{E}^i_\star, \mathcal{E}^i_B]$. Therefore, it ensures $\gamma \cdot \mathcal{E}^i_B$ lies between $\mathcal{E}^i_\star$ and $\mathcal{E}^i_B$. In our test, we can continually enlarge $\mathcal{E}^i_m$ by adjusting γ to optimize the connectivity cost and constrain the ergodicity until ESPC maintains connectivity in all periods. This design allows us to compare ESPC with the baselines more easily.

B. Baseline Methods: Periodic Connection

1) Baseline 1: This baseline differs from ESPC by enforcing (hard) connection constraints at certain time points during ergodic search. A common strategy to maintain connectivity is to navigate all robots periodically to the same predefined area with small change between the previous trajectories and the modified trajectories for connection (similar to [16]). Let $q^{cen}_{\star,\nu}(t) = \frac{1}{N}\sum_{i \in I_N} q^i_{\star,\nu}(t)$ denote the average of the positions of all robots along the ν-th dimension at time t, where $q^i_{\star,\nu}(t)$ is the ν-th dimension of position $q^i_\star(t) = g(x^i_\star(t))$ obtained by solving Problem 1, and $q_\star = (q^1_\star, q^2_\star, \cdots, q^N_\star)$ denote the joint trajectory. Baseline 1 minimizes the quadratic objective $(q - q_\star)^T \mathcal{Q}(q - q_\star)$ with a unit penalty matrix $\mathcal{Q}$ while maintaining connectivity periodically by adding the constraint $\sum_{\nu=1}^3 \left(q^i_\nu(pT_p) - q^{cen}_{\star,\nu}(pT_p)\right)^2 \leq R^2_c/4$. This constraint ensures all robots connect periodically in a sphere (related to ν) with R_c being the sphere diameter, and the sphere is centered on the geometric center of the trajectory $q^i_\star$ for all robots $i \in I_N$ at the end of each period $t = pT_p, \forall p \in I_p$. The other constraints are the same as in Problem 1. We use Alg. 2 to solve the problem, and the initial guess is set to $x_\star, u_\star$.

2) Baseline 2: Baseline 2 minimizes the sum of each robot's ergodic metric, i.e., $\sum_{i \in I_N} \mathcal{E}^i(\phi^i, q^i)$ with a periodic connection constraint. The constraint requires all robots to meet at S^{des}, a pre-defined sphere centered on $q^{des} \in \mathcal{W}$ with the diameter R_c, at periodic time points $t = pT_p, \forall p \in I_p$, i.e., $\sum_{\nu=1}^3 \left(q^i_\nu(pT_p) - q^{des}_\nu\right)^2 \leq R^2_c/4, \forall p \in I_p$. The idea is similar to [17], where connection locations are defined as sets for each robot pair (sub-team). In our tests, we set $q_{des} = [0, 0, H]$, which is the center of the workspace. The other constraints are the same as those in Problem 1, and the initial guess is set to $x(t) = x_0, t \in [0, T]$ and $u = \mathbf{0}$.

3) Baseline 3: Baseline 3 directly solves Problem 2 without using the two-phase (i.e., first solve Problem 1 and then Problem 2). Introducing this baseline method aims to verify the benefit of using two-phase optimization, and the result is shown in Sec. V-C.5. In our tests, Baseline 3 solves Problem 2 with the initial guess $x(t) = x_0, t \in [0, T], u = \mathbf{0}$.

C. Numerical Results

1) Overview: Fig. 3 shows the robots' trajectories and inter-robot distances obtained by different methods within four periods ($N_p = 4$) and four robots ($N = 4$). For the first four rows (a-t), all robots explore the common information map $\bar{\phi}$. For the last row (u-y), each robot has its information map $\phi^i, i \in I_N$ to be explored.

In Row 1 (a-d), all robots seek to plan ergodic trajectories without considering connectivity maintenance. The solution trajectories $x_\star, u_\star$ are planned by the phase 1 optimization,

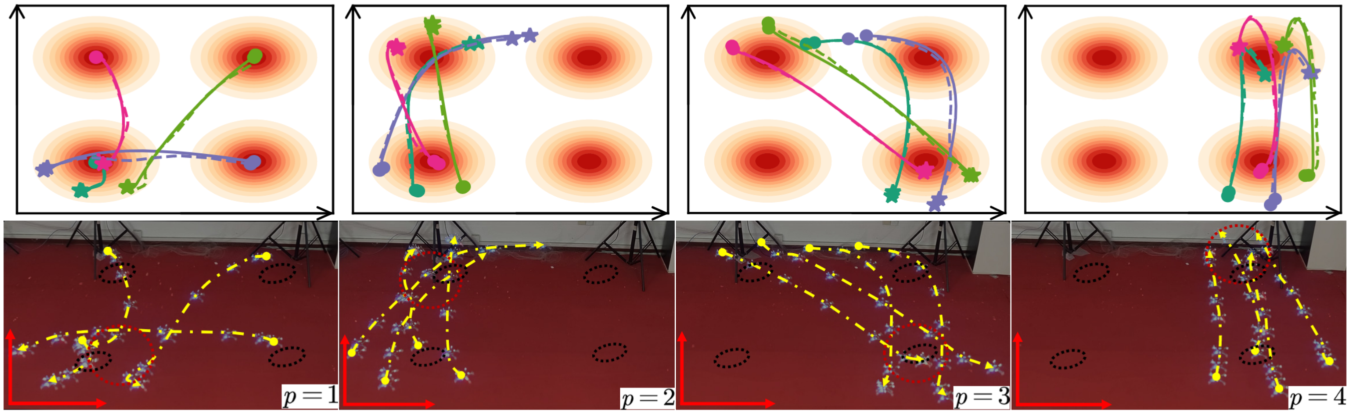


Fig. 5. The hardware experiment of ESPC with $N = 4$ and $N_p = 4$. In Row 1, the solid and dashed lines represent the planned reference trajectories and the actual trajectories. Row 2 shows the trajectories of the quadrotors.

and the inter-distance (e) is often greater than R_c, indicating poor connectivity. For Baseline 1 and 2, (j) and (o) show that the inter-distance $d_{ij}(t), i, j \in I_N, i \neq j$, is periodically less than R_c. In Row 4 (p-s), our method ESPC plans the trajectories by optimizing the ergodic metric and the connectivity cost, where the connection locations differ for each period and the inter-distance (t) is still less than R_c within each relaxed connection period. The trajectories shown in (p-s) are the solution after phase 2 optimization, seeded by $x_\star, u_\star$ as the initial guess. And the different trajectories and inter-robot distances shown in (p-t) and (a-e) demonstrate the effect of phase 2 optimization.

2) Numbers of Connection Periods: Fig. 4a and 4b shows the impact of period numbers $N_p \in \{2, 3, \cdots, 8\}$ on ergodic metric when all robots explore the common information map $\bar{\phi}$ ((a-t) in Fig. 3) or each robot has its information map $\phi^i, i \in I_N$ ((u-y) in Fig. 3). We observe that ESPC usually achieves an order of magnitude lower (i.e., better) ergodicity compared to the baselines for various N_p. Meanwhile, the ergodicity of both the baselines and ESPC is larger (i.e., worse) than only optimizing the ergodicity while ignoring the connectivity maintenance, as shown by $\mathcal{E}_\star$. It indicates that maintaining connectivity and minimizing ergodicity are often two conflicting objectives. Finally, as N_p increases, the length of each period T/N_p decreases, which means robots need to connect more frequently, and ergodicity worsens.

3) Numbers of Robot: Fig. 4c shows the ergodic metric with different N under a fixed $N_p = 4$. All robots explore the common information map as shown in Row 1-4 (a-t) of Fig. 3. First, $\mathcal{E}_\star$ (i.e., ergodic search without connectivity maintenance) decreases as N increases. However, this method fails to maintain connectivity. Meanwhile, the ergodicity obtained by the baselines $\mathcal{E}_{B1}$, $\mathcal{E}_{B2}$ and ESPC $\mathcal{E}_c$ fluctuate within a certain range. Additionally, $\mathcal{E}_c$ is often an order of magnitude smaller than $\mathcal{E}_{B1}$ and $\mathcal{E}_{B2}$ when all robots are fully connected during all periods. The result shows ESPC obtains better ergodicity than the baselines while achieving similar connectivity with varying robot numbers.

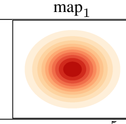
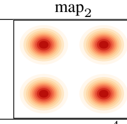
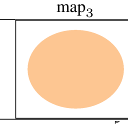
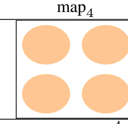
4) Various Information Maps: Here, we fixed the parameters $N_p = 4, N = 4$ and let all robots explore the common information map. Tab. I shows the four different types of information maps used in this test, and the ergodic metrics $\mathcal{E}_\star, \mathcal{E}_{B1}, \mathcal{E}_{B2}, \mathcal{E}_c$ obtained by different methods. ESPC, Baseline 1, and Baseline 2 maintain connectivity during all periods ($C_{con} = 0$), whereas ergodic search without connectivity maintenance fails, resulting in high C_{con} values of $14, 22, 22, 24$ for maps $1-4$, respectively. ESPC achieves lower ergodicity than the baselines. Besides, the information maps with geographically concentrated distributions $(\mathrm{map}_1, \mathrm{map}_3)$ exhibit lower ergodicity than dispersed ones $(\mathrm{map}_2, \mathrm{map}_4)$ since the robots can meet more easily in these areas with high information density. Note that the ergodicity varies depending on the type of information map. In map_1, $\mathcal{E}_\star$ is even slightly higher than $\mathcal{E}_c$, and the possible reason is that the phase 1 optimization can get trapped in a local minimum, which is common for ergodic search.

5) Baseline 3: This section compares ESPC against Baseline 3. The test settings are the same as in map_2 of Tab. I. With the maximum iteration $K = 50$, Baseline 3 failed to converge to a dynamically feasible trajectory after running 279.94 seconds, and the average equality error is 0.31, while ESPC achieves convergence after 133.14 seconds with an average equality error of 0.01. This result justifies the benefit of solving Problem 2 in two phases as in ESPC.

D. Simulation and Hardware Experiments

We first validate the effectiveness of our proposed ESPC planning method by simulating 8 quadrotors using Crazysim [32] (see attachment). The hardware experiment is conducted with $N = 4, N_p = 4$ using Crazyflie 2.1 [33], Crazyswarm2 [31], ROS2 [34] and a motion capture system.

TABLE I
ERGODIC METRICS WITH VARIOUS INFORMATION MAPS

Name	map_1	map_2	map_3	map_4
$\bar{\phi}$				
$\mathcal{E}_\star$	7.03×10^{-5}	1.47×10^{-4}	6.95×10^{-5}	1.07×10^{-4}
$\mathcal{E}_{B1}$	6.98×10^{-3}	6.25×10^{-2}	1.50×10^{-2}	4.98×10^{-2}
$\mathcal{E}_{B2}$	8.12×10^{-5}	3.01×10^{-2}	1.48×10^{-3}	2.27×10^{-2}
$\mathcal{E}_c$	7.00×10^{-5}	6.25×10^{-3}	6.95×10^{-5}	2.49×10^{-3}

Limited by the test space, we scale down the workspace and use the following parameters: $\mathcal{W}=[-1,1]\times[-1,1]\times[0,3]$ m^3, $R_c=0.5$ m, $\rho=6.0$, $\varepsilon_d=0.3$ m, $T=20$ s, $\delta_t=0.2$ s. All Crazyflies use PID-based position control to resist unmodeled dynamics and motion disturbances. The comparison of the reference trajectories and the actual trajectories is shown in Fig. 5, which validates that our experimental settings match the hardware, and the planned trajectories are executable on real robots in the lab. Besides, ESPC can maintain periodic connectivity during ergodic search for multiple quadrotors in a lab setting.

VI. Conclusion and Future Work

This paper studies multi-robot ergodic search with intermittent connectivity maintenance and proposes a new method, ESPC. ESPC introduces the notion of relaxed periodic connectivity and the corresponding problem formulation. To solve the problem, ESPC uses a two-phase method based on the augmented Lagrangian method. We compared ESPC against baselines in simulation and verified the results on quadrotors. The results show that ESPC can maintain connectivity while achieving low ergodic metrics. Future work includes a comparison with our other recent work [35], and distributed planning to handle numerous robots, as opposed to the centralized planning in this work.

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